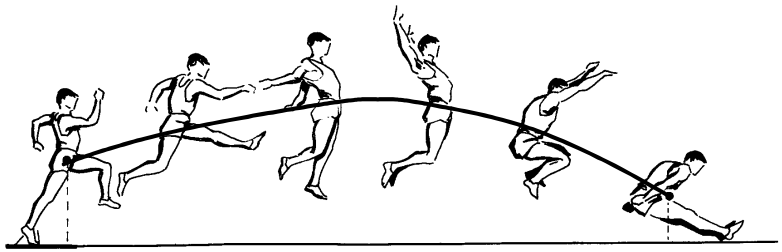
# Projectile Motion I

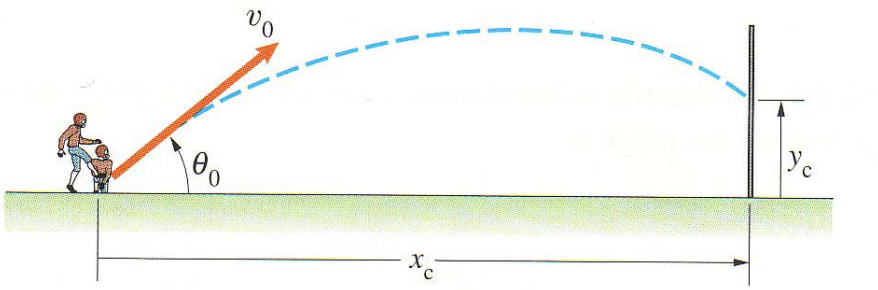
Spring 2005

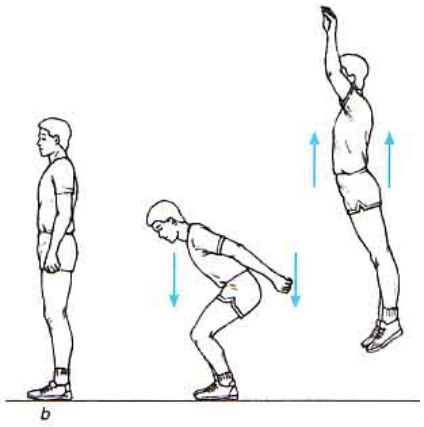
# What you should know

- What is the projectile motion?
- What are three primary factors affecting projectile motions?
- What are four governing equations?

# Projectile Motion






- Projectile : An object that has no external forces acting on it other than _____
- Uniformly accelerated motion (acceleration = g = _____ $m/s^2$)
- Motions of an object in the air.
- Long jump, high jump, goal kick, basketball throw, shotput, …….
- No air-resistance ➔ Mathematical analysis

# Take-home message……

- Different approach to horizontal direction (X-axis) and vertical direction (Y-axis)
- Horizontal motion ➔

(no air-resistance assumption)

Constant velocity (Newton's 1st law)

- Vertical motion ➔ _____ (g= -9.8 $m/s^2$ )
- _____ velocity vector determine the path of projectile motion.

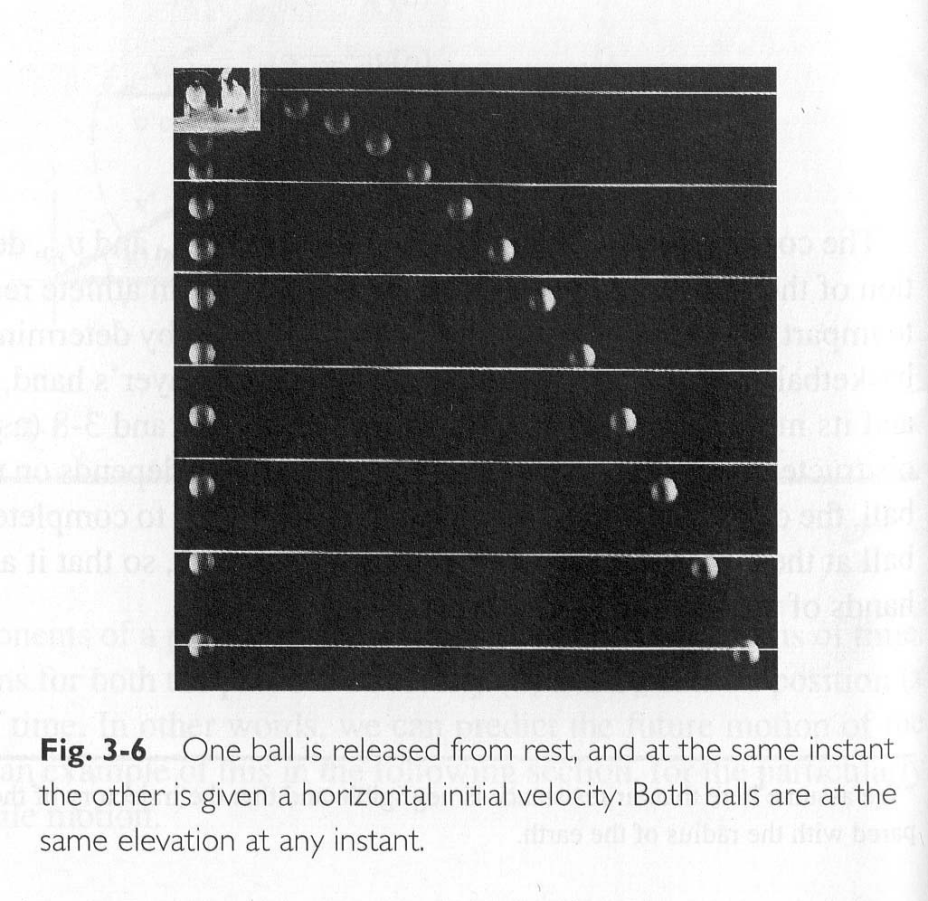
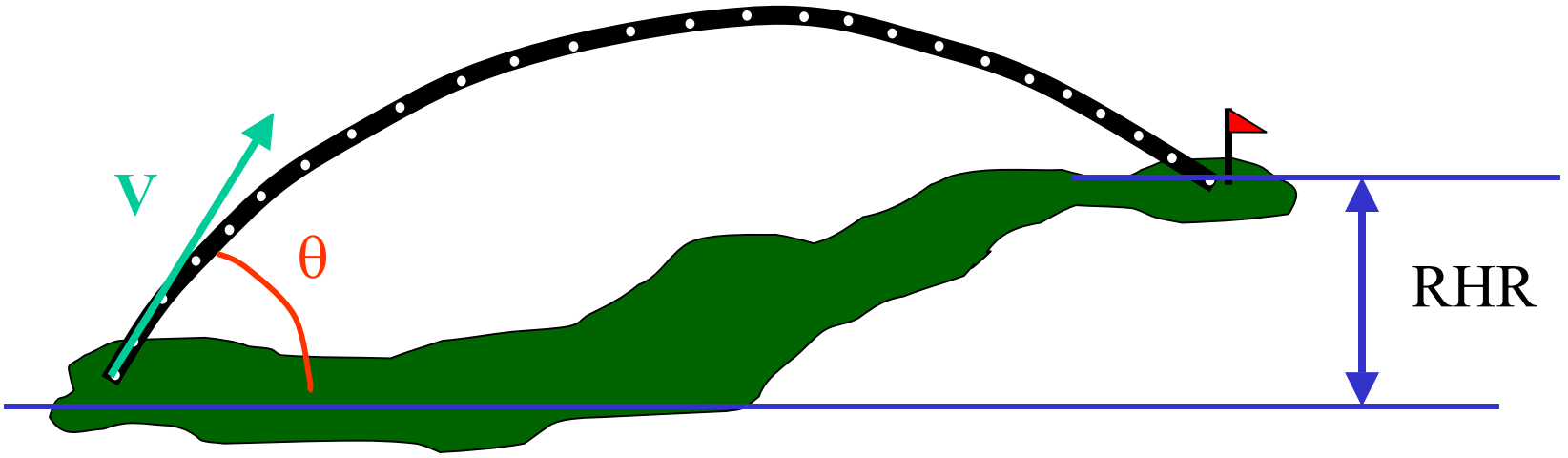


**Fig. 3-6** One ball is released from rest, and at the same instant the other is given a horizontal initial velocity. Both balls are at the same elevation at any instant.

# Primary factors affecting performance

- Angle of Release (= projection angle or take-off angle)
- Relative Height of Release (RHR)

  projection height = release height – landing height
- Speed of Release (= projection velocity or take-off velocity)



# Is a Higher angle better?

- If the goal is vertical jump height or vertical distance, what takeoff angle is best?

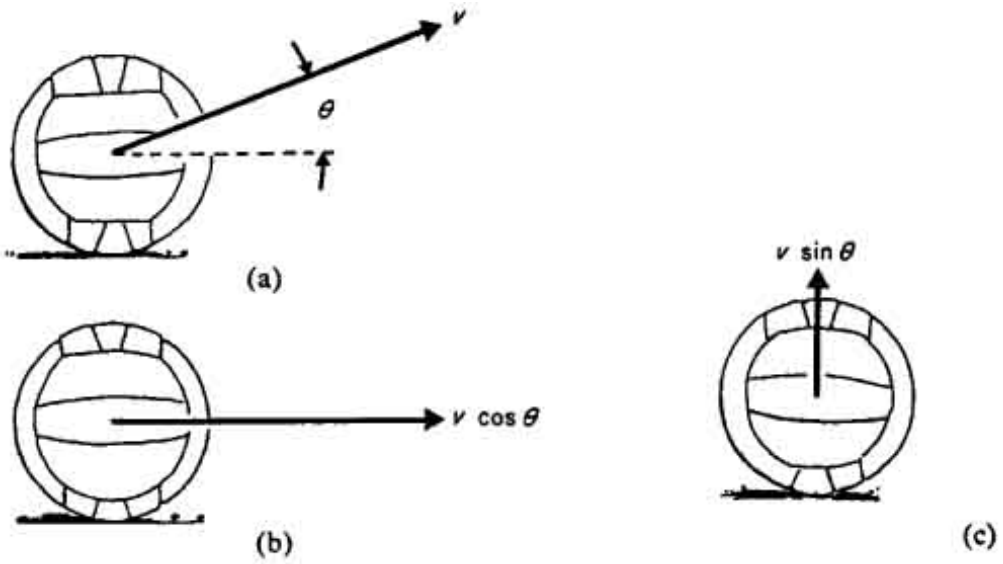
- If the goal is long jump or distance, an angle of ____ is the best (When RHR = 0). Why?

- Does this Optimal angle change if relative release height is not equal to 0?

# Angle of Release

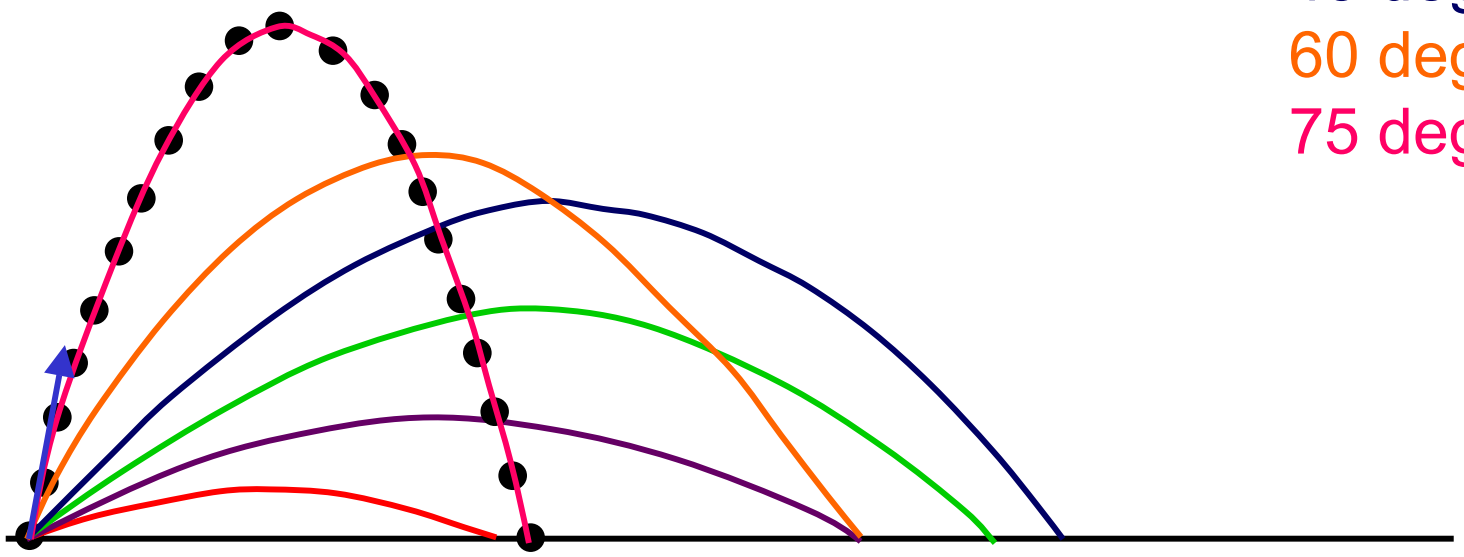
Trade-off between horizontal velocity ($V_H$) and vertical velocity ($V_V$)

As $\theta\uparrow$ ➔ $V_H$ ___ (due to cosine) , $V_V$ ___(due to sine)



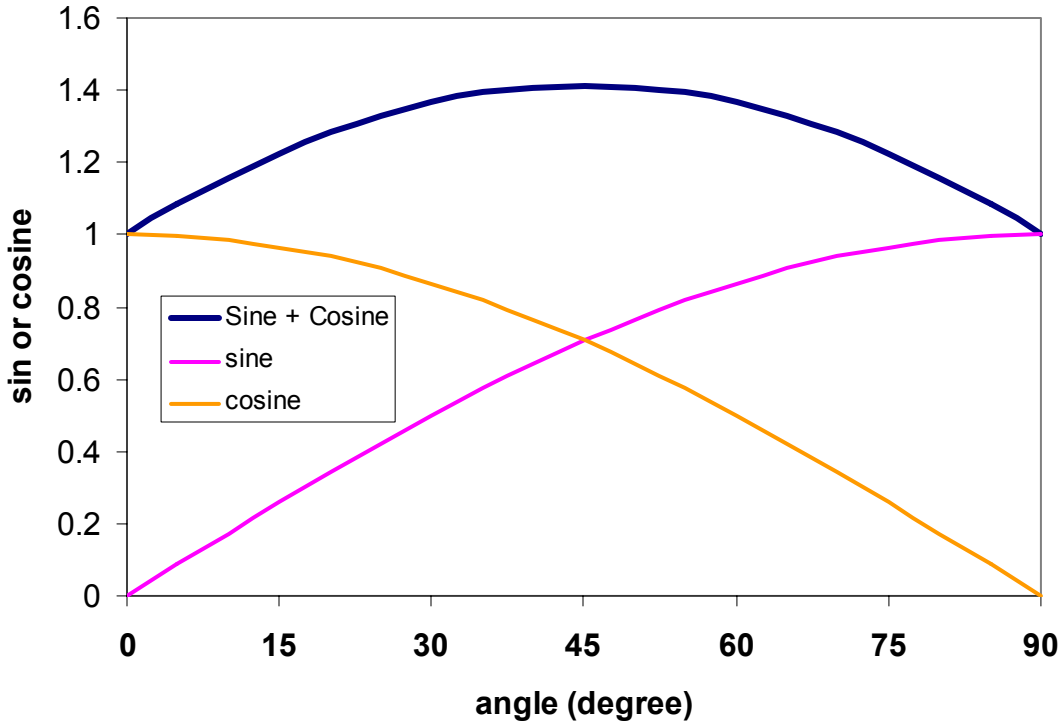
# Release angle = 75 degrees

10 degrees
30 degrees
40 degrees
45 degrees
60 degrees
75 degrees



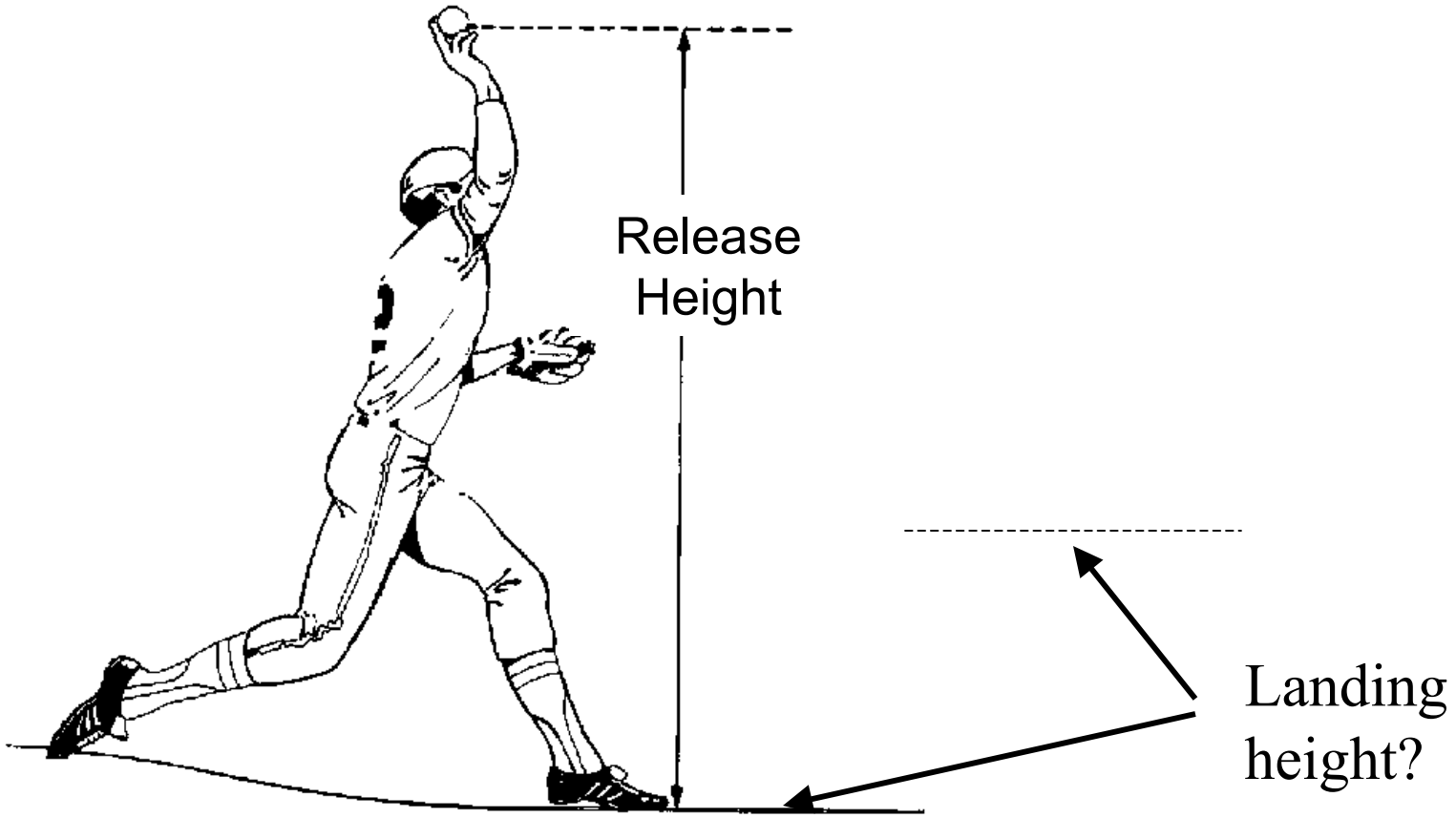
So the angle that maximizes horizontal distance appears to be ($\theta_{optimal}$) = 45 degrees

# Due to the characteristics of the trigonometric functions




↑ Angle ➔ $V_V$ increases, but $V_H$ decreases.
There is a Trade-off between $V_V$ and $V_H$

# Relative Height of Release (RHR)

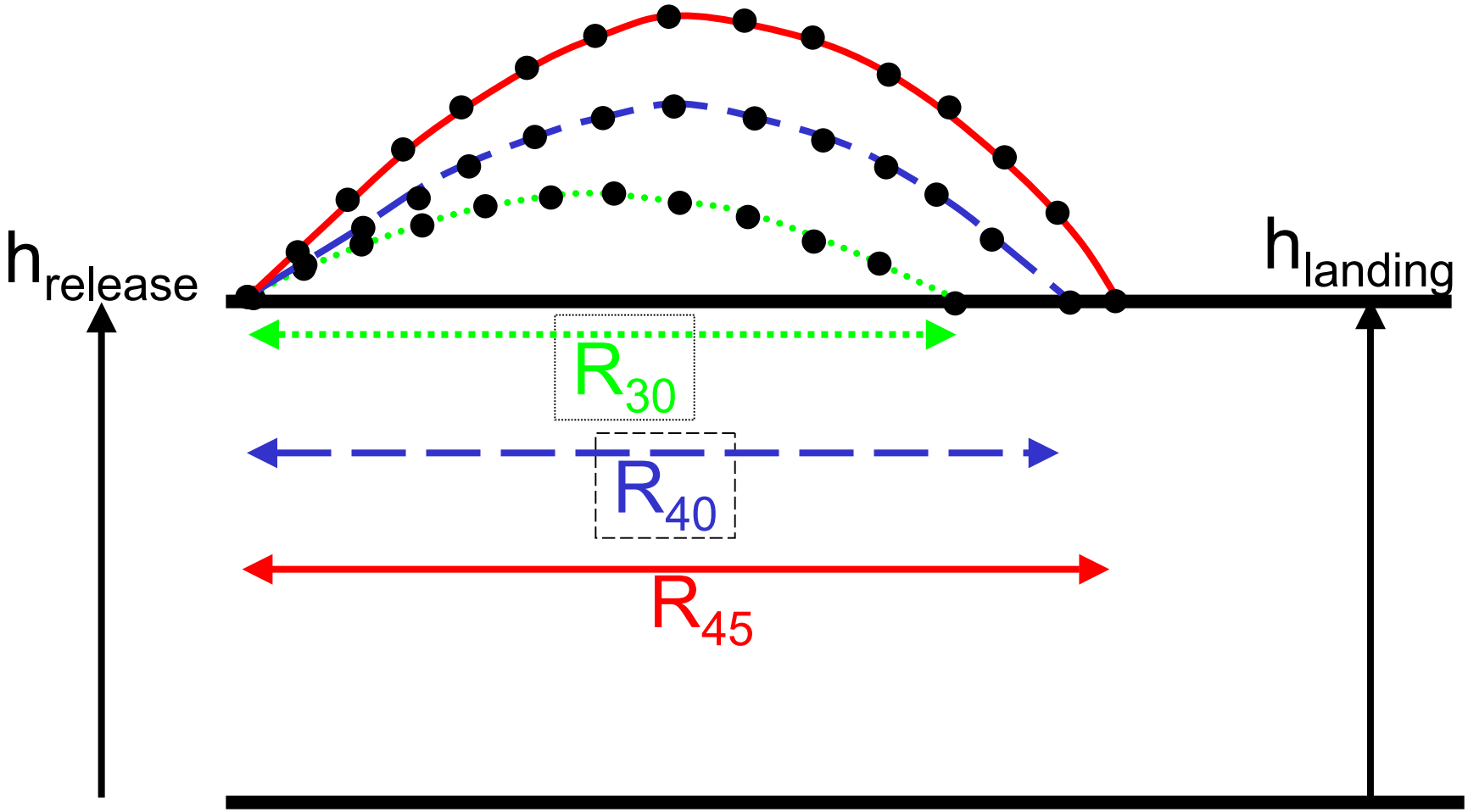



- RHR = release height - landing height

# RHR and Angle of Release interact to affect D

RHR = 0

$R_{45} > R_{40} > R_{30}$

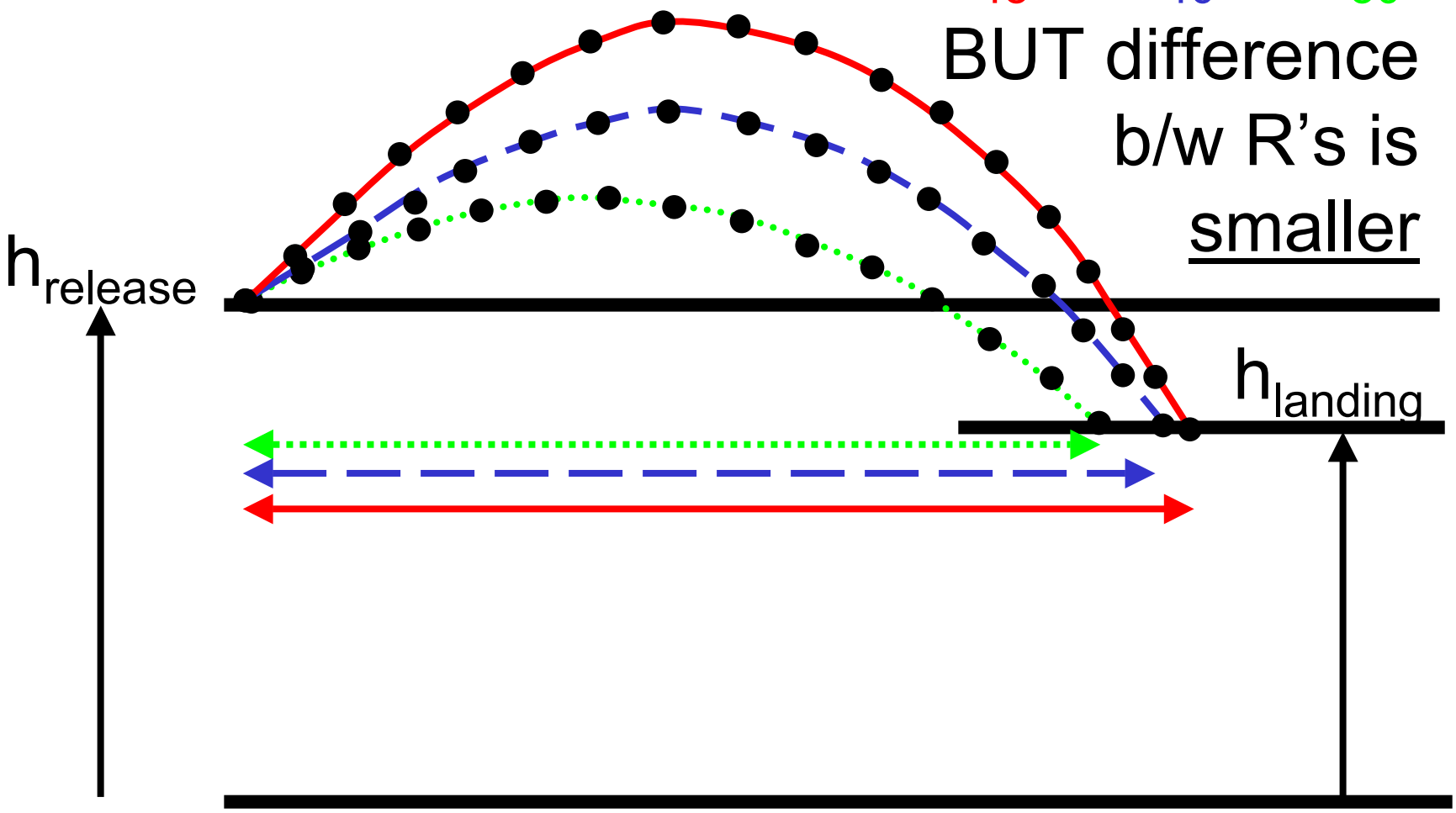


# When $h_{landing} < h_{release}$

RHR > 0

$R_{45} > R_{40} > R_{30}$

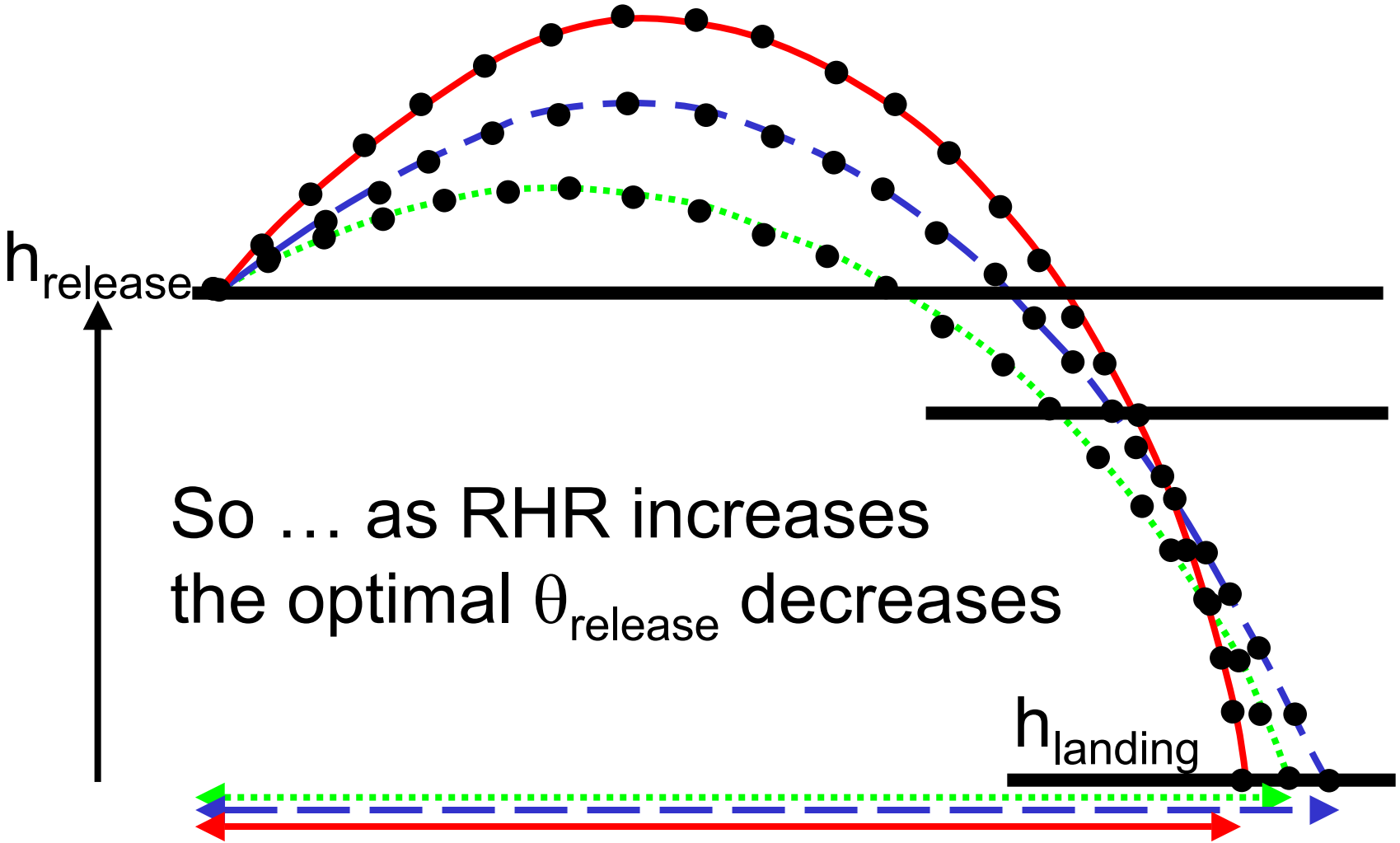
BUT difference b/w R's is <u>smaller</u>



When $h_{landing} << h_{release}$

RHR > 0

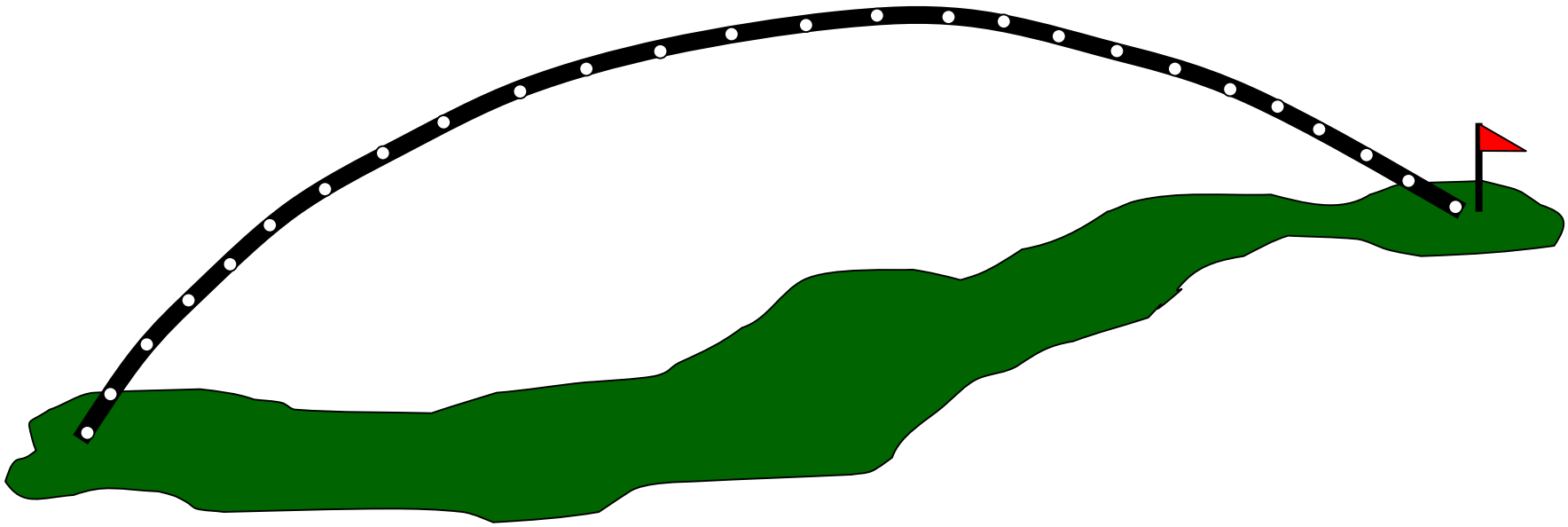
$R_{40} > R_{30} > R_{45}$



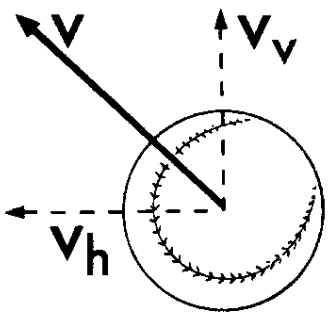

So … as RHR increases
the optimal $\theta_{release}$ decreases

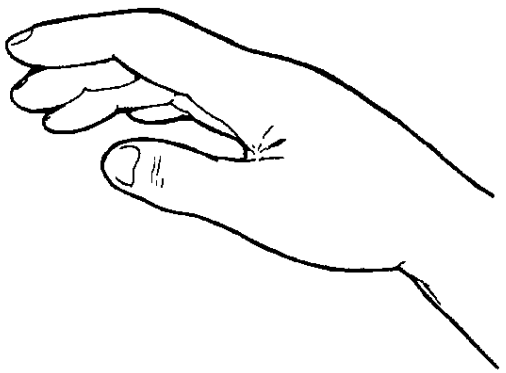
It's possible to have a *negative* RHR ($h_{release} < h_{landing}$)

In this case the optimal $\theta_{release}$ is _____ than 45 degrees



# Speed of Release



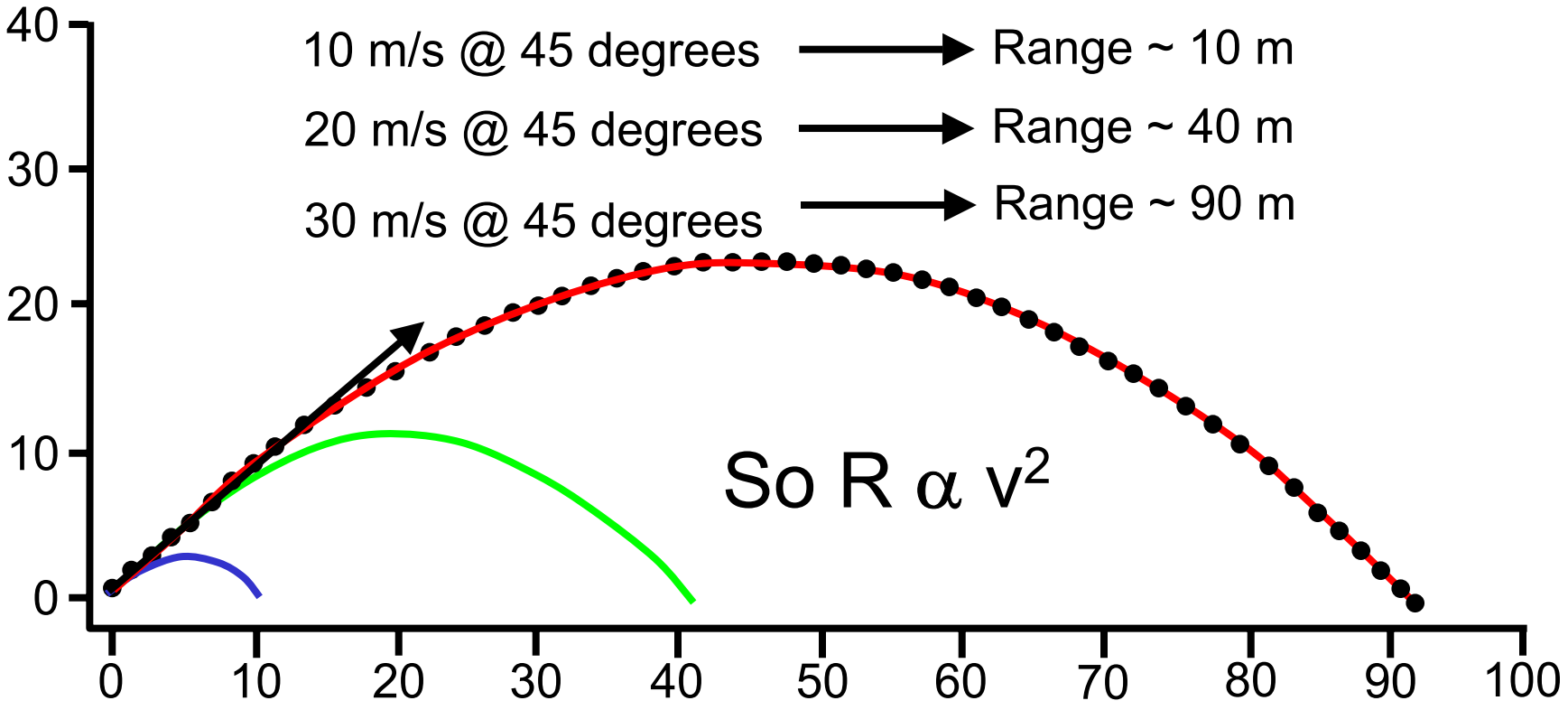
- Horizontal velocity does not change while the object is in the air.

- Vertical velocity changes by -9.8 m/s$^2$ for every second the object is in the air.

$$R = \frac{v^2 \sin\theta\cos\theta + v\cos\theta\sqrt{(v\sin\theta)^2 + 2gh}}{g}$$

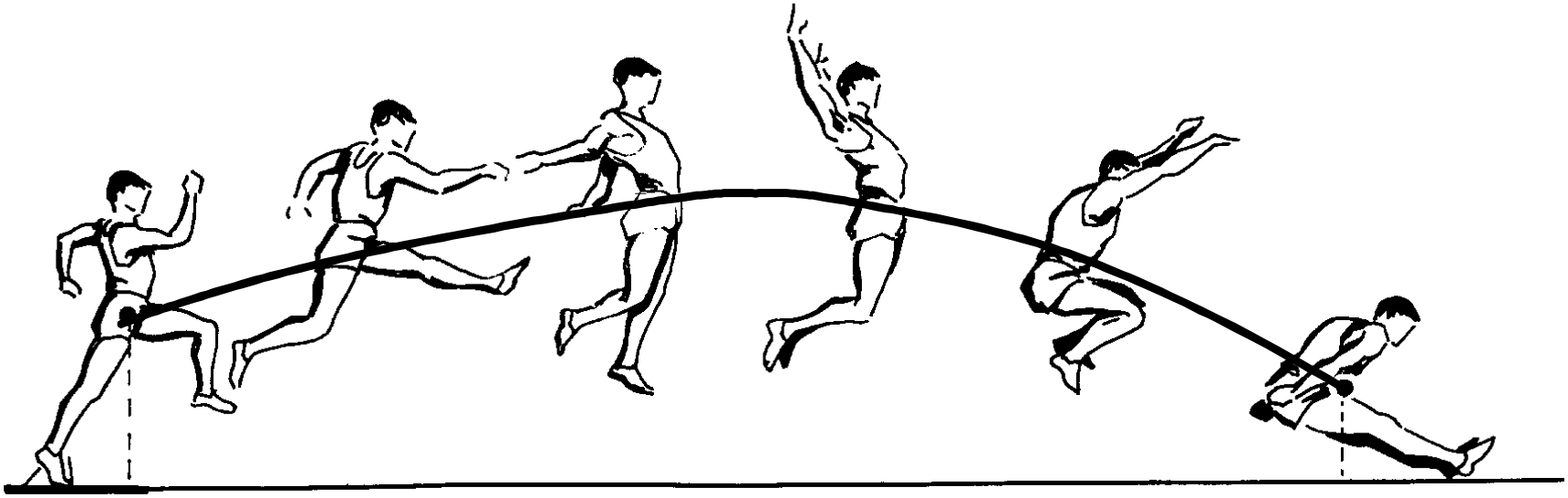
- Because R $\alpha$ v$^2$, it has the greatest influence on the horizontal range of the projectile

# The effect of Speed of Release on the horizontal range of a projectile



# Long Jump

- What is the optimum angle of takeoff for long jumpers?



- RHR > 0 (take-off height > landing height)
- Optimum Angle should be slightly less than 45 degrees
- Research shows that it should be 42-43 degrees

| *Athlete* | *Distance of Jump Analyzed (m)* | *Speed of Takeoff (m/s)* | *Optimum Angle of Takeoff for Given Speed (deg)* | *Actual Angle of Takeoff (deg)* |
|---|---|---|---|---|
| Mike Powell (USA) | 8.95 | 9.8 | 43.3 | 23.2 |
| Bob Beamon (USA) | 8.90 | 9.6 | 43.3 | 24.0 |
| Carl Lewis (USA) | 8.79 | 10.0 | 43.4 | 18.7 |
| Ralph Boston (USA) | 8.28 | 9.5 | 43.2 | 19.8 |
| Igor Ter-Ovanesian (USSR) | 8.19 | 9.3 | 43.2 | 21.2 |
| Jesse Owens (USA) | 8.13 | 9.2 | 43.1 | 22.0 |
| Elena Belevskaya (USSR) | 7.14 | 8.9 | 43.0 | 19.6 |
| Heike Dreschler (GDR) | 7.13 | 9.4 | 43.2 | 15.6 |
| Jackie Joyner-Kersee (USA) | 7.12 | 8.5 | 42.8 | 22.1 |
| Anisoara Stanciu (Rom) | 6.96 | 8.6 | 42.9 | 20.6 |
| Vali Ionescu (Rom) | 6.81 | 8.9 | 43.0 | 18.9 |
| Sue Hearnshaw (GB) | 6.75 | 8.6 | 42.9 | 18.9 |

Actual Angle of Takeoff ~ 17-23 degrees

# Long Jump

- When a jumper is moving at 10 m/s
  - the foot is not on the ground long enough to generate a large takeoff angle
  - so jumpers maintain speed and live with a low takeoff angle

- **v is the most important factor in projectile motion, why?**

# VALUES FOR HYPOTHETICAL JUMPS UNDER DIFFERENT CONDITIONS

| Variable | Values for Actual Jump (1) | Speed of Takeoff Increased 5% (2) | Angle of Takeoff Increased 5% (3) | Relative Height of Takeoff Increased 5% (4) |
|---|---|---|---|---|
| Speed of Takeoff | 8.90 m/s | 9.35 m/s | 8.90 m/s | 8.90 m/s |
| Angle of Takeoff | 20 | 20 | 21 | 20 |
| Relative Ht of Takeoff | 0.45 m | 0.45 m | 0.45 m | 0.47 m |
| Horizontal Range | 6.23 m | 6.77 m | 6.39 m | 6.27 m |
| Change in Horiz Range | -- | **0.54 m** | **0.16 m** | **0.04 m** |
| Distance of Jump | 7.00 m | 7.54 m | 7.16 m | 7.04 m |